CIRCUIT DESIGNER'S NOTEBOOK

Effective Capacitance vs Frequency

It is generally assumed that the capacitance value selected from a vendor's catalog is constant over frequency. This is essentially true for applications with applied frequencies that are well below the capacitors self-resonant frequency. However as the operating frequency approaches the capacitors self-resonant frequency, the capacitance value will appear to increase resulting in an effective capacitance (C_E) that is larger than the nominal capacitance. This article will address the details of effective capacitance as a function of the application operating frequency. In order to illustrate this phenomenon, a simplified lumped element model of a capacitor connected to a frequency source operating in a network will be considered, as depicted in Figure 1.

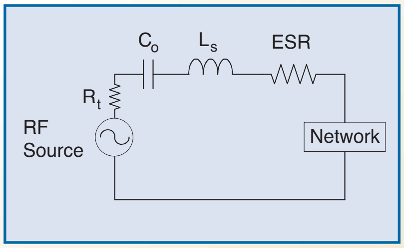

Figure 1
Lumped Element Equivalent Model

This model has been selected because the effective capacitance is largely a function of the net reactance developed between the capacitor and its parasitic series inductance (L_S). The equivalent series resistance 'ESR' shown in this illustration does not have a significant effect on the effective capacitance.

Effective Capacitance:

The nominal capacitance value (C_O) is established by a measurement performed at 1MHz. In typical RF applications the applied frequency is generally much higher than the 1MHz measurement frequency, hence at these frequencies the inductive reactance (X_L) associated with the parasitic series inductance (L_S) becomes significantly large as compared to the capacitive reactance (X_C). Figure 2 illustrates that there is a disproportionate increase in X_L as compared to X_C with increasing frequencies. This results in an effective capacitance that is greater than the nominal capacitance. Finally at the capacitors series resonant frequency the two reactance's are equal and opposite yielding a net reactance of zero. The expression for C_E becomes undefined at this frequency.

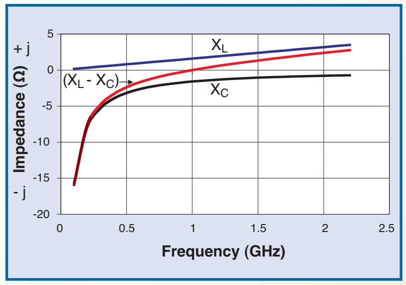

Figure 2
Net Impedance vs. Frequency

As illustrated in Figure1, the physical capacitor can be represented as C_O in series with L_S. The impedance of the series combination of C_O and L_S can then be set equal to C_E, which may be referred to as an "ideal equivalent" capacitor. This will yield the following equation:

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$$j(\omega L_S - 1/\omega C_0) = -j\,1/\omega C_E$$
$$\omega^2 L_S - 1/C_0 = -1/C_E$$

The relationship between the operating frequency F_0 and the effective capacitance C_E can then be stated as:

$$C_E = C_0/(1 - \omega^2 L_S C_0)$$
$$C_E = C_0/[1 - (2\pi F_0)^2 L_S C_0]$$

Where:

C_E = Effective Capacitance at the application frequency, (F_0)
C_0 = Nominal Capacitance at 1 MHz
L_S = Parasitic Inductance, (H)
F_0 = Operating Frequency, (Hz)

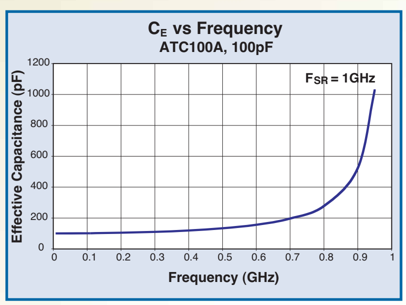

Figure 3
Effective Capacitance (C_E) vs. Frequency

From this relationship it can be seen that as the applied frequency increases the denominator becomes smaller thereby yielding a larger effective capacitance. At the capacitors series resonant frequency the denominator goes to zero and the expression becomes undefined. The relationship of C_E vs frequency is a hyperbolic function as illustrated in Figure 3.

Example:

Consider an ATC 100A series 100pF capacitor.

Calculate the effective capacitance (C_E) at 10MHz, 100MHz, 500MHz, 900MHz, 950MHz.

Solution: Calculate by using the relationship $C_E = C_0/[1-(2\pi F_0)^2 L_S C_0]$. Refer to Table 1.

Operating Frequency (MHz)	Effective Capacitance (C_E), pF	Impedance, (Ω)
10	100.01	0.013 – j 159.13
100	101.01	0.023 – j 15.76
500	133.34	0.051 – j 2.38
900	526.29	0.069 – j 0.337
950	1025.53	0.070 – j 0.168

Table 1
Relationship between F_0, C_E and Z

Application Considerations:

Impedance matching and minimum drift applications such as filters and oscillators require special attention regarding C_E. For applications below the capacitors self-resonant frequency the net impedance will be capacitive (-j) whereas for applied frequencies above resonance the net impedance will be inductive (+j). Operating above series resonance will correspondingly place the impedance of the capacitor on the inductive side of the Smith chart (+j). When designing for these applications both C_E and the sign of the net impedance at the operating frequency must be carefully considered.

In contrast, the majority of coupling, bypass and DC blocking applications are usually not sensitive to the sign of the impedance and can be capacitive or inductive, as long as the magnitude of the impedance is low at the applied frequency. The effective capacitance will be very large and the net impedance will be very low when operating close to resonance. At resonance the net impedance will be equal the magnitude of ESR and the capacitance will be undefined.

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Excerpt from complete Circuit Designers' Notebook, Document #001-927, Rev. E, 1/05